

CS 465

AES

Programming Lab #1

- Implement AES
- Use the FIPS 197 spec as your guide
 - Avoid looking at code on the Internet
 - Challenge yourself to implement the algorithm based on sources mentioned in the lab specification
 - The standard provides programming language independent pseudo-code
 - 20 pages in the spec of complete, step by step debugging information to check your solution

Finite Fields

- AES uses the finite field $GF(2^8)$
 - $b_7x^7 + b_6x^6 + b_5x^5 + b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0$
 - $\{b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0\}$
- Byte notation for the element: $x^6 + x^5 + x + 1$
 - $0x^7 + 1x^6 + 1x^5 + 0x^4 + 0x^3 + 0x^2 + 1x + 1$
 - $\{01100011\}$ – binary
 - $\{63\}$ – hex
- Has its own arithmetic operations
 - Addition
 - Multiplication

Finite Field Arithmetic

- Addition (XOR)
 - $(x^6 + x^4 + x^2 + x + 1) + (x^7 + x + 1) = x^7 + x^6 + x^4 + x^2$
 - $\{01010111\} \oplus \{10000011\} = \{11010100\}$
 - $\{57\} \oplus \{83\} = \{d4\}$
- Multiplication is tricky

Finite Field Multiplication (•)

These cancel out

$$(x^6 + x^4 + x^2 + x + 1)(x^7 + x + 1) =$$

$$x^{13} + x^{11} + x^9 + x^8 + x^7 + x^7 + x^5 + x^3 + x^2 + x + x^6 + x^4 + x^2 + x + 1$$

$$= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1$$

and

$$x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \text{ modulo } (x^8 + x^4 + x^3 + x + 1) \\ = x^7 + x^6 + 1.$$

Irreducible
Polynomial

Efficient Finite Field Multiply

- There's a better way
 - `xtime()` – very efficiently multiplies its input by $\{02\}$
 - This is the same as multiplying a polynomial by x
- Multiplication by higher powers can be accomplished through repeated applications of `xtime()`

Efficient Finite Field Multiply

Example: $\{57\} \cdot \{13\}$

$$\{57\} \cdot \{02\} = \text{xtime}(\{57\}) = \{ae\}$$

$$\{57\} \cdot \{04\} = \text{xtime}(\{ae\}) = \{47\}$$

$$\{57\} \cdot \{08\} = \text{xtime}(\{47\}) = \{8e\}$$

$$\{57\} \cdot \{10\} = \text{xtime}(\{8e\}) = \{07\}$$

$$\begin{aligned} \{57\} \cdot \{13\} &= \{57\} \cdot (\{01\} \oplus \{02\} \oplus \{10\}) \\ &= \{57\} \cdot (\{01\} \oplus \{02\} \oplus \{10\}) \\ &= (\{57\} \cdot \{01\}) \oplus (\{57\} \cdot \{02\}) \oplus (\{57\} \cdot \{10\}) \\ &= \{57\} \oplus \{ae\} \oplus \{07\} \\ &= \{fe\} \end{aligned}$$

AES Parameters

- **Nb** – Number of columns in the State
 - For AES, Nb = 4
- **Nk** – Number of 32-bit words in the Key
 - For AES, Nk = 4, 6, or 8
- **Nr** – Number of rounds (function of Nb and Nk)
 - For AES, Nr = 10, 12, or 14

AES methods

- Convert to state array
- Transformations (and their inverses)
 - AddRoundKey
 - SubBytes
 - ShiftRows
 - MixColumns
- Key Expansion

Inner Workings

- See Flash demo URL on course **Lectures** pages